Asymmetric Orbifold Models of Non-supersymmetric Heterotic Strings

Toshihiro Sasada

Graduate School of Science and Technology

Kobe University

Rokkodai, Nada, Kobe 657, Japan

Abstract

We investigate asymmetric orbifold models constructed from non-supersymmetric heterotic strings. We systematically classify the asymmetric orbifold models with standard embeddings and present a list of asymmetric orbifolds which are geometrically interpreted as toroidal compactifications of non-supersymmetric heterotic strings. By studying non-standard embedding models, we also construct examples of the *supersymmetric* asymmetric orbifold models based on non-supersymmetric heterotic strings.

1 Introduction

Asymmetric orbifolds [1] belong to the efficient construction schemes of 4D string models. In constructing asymmetric orbifolds, we consider the left-right asymmetric twists of the momentum lattices of toroidal compactifications. Thus, we have in general no direct geometric interpretation of asymmetric orbifolds. However, some class of asymmetric orbifolds may possess the geometric interpretation through the quantum equivalence to other compactifications. In the previous paper [2], we have studied the geometric interpretation of asymmetric orbifolds as Narain's toroidal compactifications [3] and found a simple condition for the geometric interpretation of asymmetric orbifolds.

In this paper we construct the asymmetric orbifold models which are based on the non-supersymmetric heterotic strings [4], and investigate the geometric interpretation of such asymmetric orbifolds as the toroidal compactification [5] of non-supersymmetric heterotic strings. Asymmetric orbifold models of non-supersymmetric heterotic strings have not been studied before. The main purpose of this paper is to extend the previous analysis by studying the geometric interpretation of the *non-supersymmetric* asymmetric orbifold models. We classify the asymmetric orbifold models with standard embeddings in a systematic way and give a list of the asymmetric orbifold models which are geometrically interpreted as toroidal compactifications of non-supersymmetric heterotic strings.

We shall study the condition for interpreting orbifold models as toroidally compactified ones. A crucial feature of ten-dimensional heterotic strings is the existence of NSR fermions. They will form an $SO(8)$ Kač-Moody algebra in the light-cone gauge. Since the compactification onto tori yields no effect on the NSR fermions, the fermions on orbifolds should generate $SO(8)$ Kač-Moody algabras if they are interpreted as toroidal compactifications. We define fermion currents as the current operators with conformal weight one which generate $SO(N)$ algebra and which contain the $SO(2)$ Kač-Moody algebra generated by the space-time component of the fermions on orbifold models. A necessary condition for interpreting orbifold models as toroidally compactified ones is that the fermion currrents should generate $SO(8)$ Kač-Moody algebras. We shall show that the above condition is necessary and also sufficient for our class of asymmetric orbifolds. We should note that symmetric orbifolds cannot be interpreted as toroidal

compactifications since fermion currents on symmetric orbifolds are constructed by the currents in the untwisted sector which are always smaller than $SO(8)$. Orbifold models must possess twist-untwist intertwining currents [6, 7] in order to possess $SO(8)$ fermion currents. Thereby, orbifold models must be asymmetric if they are interpreted as toroidal compactifications.

The above asymmetric orbifold models with standard embeddings possess no supersymmetry. At first sight, any orbifold model constructed from non-supersymmetric heterotic strings seems to possess no supersymmetry. However, this will turn out not to be true for generic asymmetric orbifold models. In fact, for asymmetric orbifold models of supersymmetric heterotic strings supersymmetry can appear from the twisted sectors [7, 8]. We may expect that the similar mechanism still holds for some classes of the asymmetric orbifolds constructed from non-supersymmetric heterotic strings. In this paper, by studying non-standard embedding models, we shall show some examples of *supersymmetric* asymmetric orbifold models constructed from non-supersymmetric heterotic strings.

The organization of this paper is as follows. In section 2, we explain the construction of the asymmetric orbifolds from non-supersymmetric heterotic strings. In section 3, we clarify a necessary and sufficient condition for the asymmetric orbifold models to be interpreted as toroidally compactified models. In section 4, we classify the asymmetric orbifold models and investigate fermion currents on them. In section 5, we construct the examples of supersymmetric asymmetric orbifolds of non-supersymmetric heterotic strings. In section 6, we present our conclusion.

2　Asymmetric orbifold models

We shall recall the basic set up of the non-supersymmetric ten-dimensional heterotic strings [4]. The non-supersymmetric heterotic strings with rank 16 gauge groups are specified by the left-moving 16-dimensional conjugacy classes $\Gamma_i^{16,0}$ ($i = 1, \ldots, 4$). The conjugacy class $\Gamma_1^{16,0}$ ($\Gamma_2^{16,0}$) is connected to the NS sector states with the even (odd) G-parity. The conjugacy class $\Gamma_3^{16,0}$ ($\Gamma_4^{16,0}$) is connected to the R sector states with the positive (negative) chirality. For definiteness, we will concentrate on the tachyon-free non-supersymmetric heterotic strings with $SO(16) \times SO(16)$ gauge groups. Then $\Gamma_i^{16,0}$

$(i = 1, \ldots, 4)$ are given by $\Gamma_1^{16,0} = (0,0) \cup (c,c)$, $\Gamma_2^{16,0} = (s,v) \cup (v,s)$, $\Gamma_3^{16,0} = (v,v) \cup (s,s)$ and $\Gamma_4^{16,0} = (c,0) \cup (0,c)$, where 0, v, s and c are the adjoint, vector, spinor and conjugate spinor conjugacy classes of $SO(16)$, respectively. Almost arguments in this paper will be straightforwardly applied to the other non-supersymmetric heterotic strings.

Let us now construct asymmetric $\mathbf{Z}_N$-orbifolds of the non-supersymmetric heterotic string. We shall start with the toroidal compactification [5] of the non-supersymmetric heterotic string defined by the conjugacy classes $\Gamma_i^{16,0} \oplus \Gamma^{6,6}$ $(i = 1, \ldots, 4)$, where $\Gamma^{6,6}$ is a (6+6)-dimensional Lorentzian even self-dual lattice. The left- and right-moving momentum (p_L^I, p_L^i, p_R^i) $(I = 1, \ldots, 16; i = 1, \ldots, 6)$ lies on the conjugacy classes $\Gamma_i^{16,0} \oplus \Gamma^{6,6}$. The group element g which generates a cyclic group $\mathbf{Z}_N$ is defined by the following action on the string coordinates:

$$g : (X_L^I, X_L^i, X_R^i) \rightarrow (X_L^I + 2\pi v_L^I, U_L^{ij} X_L^j, U_R^{ij} X_R^j), \tag{2.1}$$

where U_L and U_R are rotation matrices which satisfy $U_L^N = U_R^N = \mathbf{1}$ and v_L^I is a shift vector. The rotation matrices U_L and U_R must be an automorphism of $\Gamma^{6,6}$:

$$(U_L^{ij} p_L^j, U_R^{ij} p_R^j) \in \Gamma^{6,6} \quad \text{for all } (p_L^i, p_R^i) \in \Gamma^{6,6}. \tag{2.2}$$

The action of the operator g on the right-moving fermions is given by the U_R rotation. Let N_ℓ be the minimum positive integer such that $(g^\ell)^{N_\ell} = 1$ in the g^ℓ-twisted sector. We shall denote the eigenvalues of U_L^ℓ and U_R by $\{e^{i2\pi\zeta_\ell^a}, e^{-i2\pi\zeta_\ell^a}; a = 1, 2, 3\}$ and $\{e^{i2\pi\zeta_R^a}, e^{-i2\pi\zeta_R^a}; a = 1, 2, 3\}$, respectively. Then we have the level matching condition for the one-loop modular invariance for N_ℓ odd

$$N_\ell \left[\frac{1}{2}(\ell v_L^I)^2 + \frac{1}{2} \sum_{a=1}^{3} \zeta_\ell^a (1 - \zeta_\ell^a) \right] = 0 \quad \text{mod } 1, \tag{2.3}$$

$$N_\ell \sum_{a=1}^{3} \ell \zeta_R^a = 0 \quad \text{mod } 2; \tag{2.4}$$

for N_ℓ even, in addition to the above conditions, we have

$$p_L^i (U_L^{\ell \frac{N_\ell}{2}})^{ij} p_L^j - p_R^i (U_R^{\ell \frac{N_\ell}{2}})^{ij} p_R^j = 0 \quad \text{mod } 2 \tag{2.5}$$

for all $(p_L^i, p_R^i) \in \Gamma^{6,6}$.

3 Torus-orbifold equivalence of heterotic strings

We now show that the necessary condition for the equivalence with torus compactifications in terms of the fermion currents is also a sufficient condition for our class of asymmetric orbifold models. As we shall see in the next section, the $\mathbf{Z}_N$-transformation of our asymmetric orbifold model is an inner automorphism of the momentum lattice. Then, the $\mathbf{Z}_N$-transformation on the lattice $\Gamma^{6,6}$ is equivalent to a shift [1, 7, 9, 10]. We shall use the bosonized representation of world-sheet fermions. The momentum p_R^t ($t = 1, \ldots, 4$) of the fermions lies on the weight lattice of $SO(8)$. The momentum in the vector (adjoint) conjugacy class corresponds to the state in the NS sector with even (odd) G-parity. The momentum in the spinor (conjugate spinor) conjugacy class corresponds to the state in the R sector with positive (negative) chirality. The $\mathbf{Z}_N$-transformation g acts on the bosons as a shift, where the shift vector is given by $v_R^t = (\zeta_R^a, 0)$. Let us denote the conjugacy classes of $SO(2n)$ as $(i)_n$ ($i = 0, v, s, c$). The momentum $(p_L'^I, p_L'^i, p_R'^i, p_R'^t)$ ($I = 1, \ldots, 16; i = 1, \ldots, 6; t = 1, \ldots, 4$) in the g^ℓ-sector ($\ell = 0$ for untwisted sector and $\ell = 1, \ldots, N - 1$ for twisted sectors) of the asymmetric orbifolds lies on the following lattice:

$$[(\Gamma_1^{16,0} \oplus \Gamma^{6,6} \oplus (v)_4) \cup (\Gamma_2^{16,0} \oplus \Gamma^{6,6} \oplus (0)_4)] + \ell(v_L^I, v_L^i, v_R^i, v_R^t), \tag{3.1}$$

for NS sector, and

$$[(\Gamma_3^{16,0} \oplus \Gamma^{6,6} \oplus (s)_4) \cup (\Gamma_4^{16,0} \oplus \Gamma^{6,6} \oplus (c)_4)] + \ell(v_L^I, v_L^i, v_R^i, v_R^t), \tag{3.2}$$

for R sector. The operator g in the g^ℓ-sector will be expressed as

$$g = \eta_\ell \exp[i2\pi(p_L'^I v_L^I + p_L'^i v_L^i - p_R'^i v_R^i - p_R'^t v_R^t)], \tag{3.3}$$

where η_ℓ is a constant phase and (v_L^i, v_R^i) is a shift vector which satisfy $N(v_L^i, v_R^i) \in \Gamma^{6,6}$. The phase η_ℓ is determined from the modular transformations [7]:

$$\eta_\ell = \exp\{-i\pi\ell[(v_L^I)^2 + (v_L^i)^2 - (v_R^i)^2 - (v_R^t)^2]\}. \tag{3.4}$$

Every physical state in the g^ℓ-sector must satisfy the condition $g = 1$ because it must be invariant under the $\mathbf{Z}_N$-transformation.

In order to examine Kač-Moody algebras on heterotic string models, it may be convenient to use the bosonic string map [11] and investigate Kač-Moody algebras on the corresponding bosonic string models. We first decompose the momentum lattices of the asymmetric orbifolds with respect to $SO(2)$ conjugacy classes to which the momentum $p_R'^t$ ($t = 4$) belongs. Then, preserving the modular transformation properties, the $SO(2)$ conjugacy classes $(i)_1$ ($i = 0, v, s, c$) are mapped to the $SO(10) \times E_8$ conjugacy classes $(i)_5 \oplus \Gamma^{0,8}$ ($i = 0, v, s, c$) as follows: $(0)_1 \to (v)_5 \oplus \Gamma^{0,8}$, $(v)_1 \to (0)_5 \oplus \Gamma^{0,8}$, $(s)_1 \to (s)_5 \oplus \Gamma^{0,8}$ and $(c)_1 \to (c)_5 \oplus \Gamma^{0,8}$, where $\Gamma^{0,8}$ is a root lattice of E_8. After the bosonic string map, the momentum $(p_L'^I, p_L'^i, p_R'^i, p_R'^t)$ in the g^ℓ-sector lies on the following lattice:

$$[(\Gamma_1^{16,0} \oplus \Gamma^{6,6} \oplus (0)_8 \oplus \Gamma^{0,8}) \cup (\Gamma_2^{16,0} \oplus \Gamma^{6,6} \oplus (v)_8 \oplus \Gamma^{0,8}) \quad (3.5)$$

$$\cup \quad (\Gamma_3^{16,0} \oplus \Gamma^{6,6} \oplus (s)_8 \oplus \Gamma^{0,8}) \cup (\Gamma_4^{16,0} \oplus \Gamma^{6,6} \oplus (c)_8 \oplus \Gamma^{0,8})] + \ell(v_L^I, v_L^i, v_R^i, v_R^t),$$

where $p_R'^t$ ($t = 4, \ldots, 8; 9, \ldots, 16$) are defined as the momentum which belong to the $SO(10) \times E_8$ conjugacy classes and $v_R^t = (\zeta_R^a, 0^5; 0^8)$. Therefore, on the corresponding bosonic strings, the momentum $(p_L'^I, p_L'^i, p_R'^i, p_R'^t)$ of the physical states in the g^ℓ-sector lies on a $(22+22)$-dimensional Lorentzian even self-dual lattice $\Gamma'^{22,22}$, where $(p_L'^I, p_L'^i, p_R'^i, p_R'^t)$ lies on the above lattice and satisfies the physical state condition:

$$p_L'^I v_L^I + p_L'^i v_L'^i - p_R'^i v_R'^i - p_R'^t v_R^t - \frac{1}{2}\ell[(v_L^I)^2 + (v_L^i)^2 - (v_R^i)^2 - (v_R^t)^2] = 0 \quad \text{mod } 1. \quad (3.6)$$

If the fermion currents on asymmetric orbifolds generate $SO(8)$ Kač-Moody algebras, then the corresponding $(22 + 22)$-dimensional lattice $\Gamma'^{22,22}$ is decomposed as follows:

$$\Gamma'^{22,22} = [(\Gamma_1'^{22,6} \oplus (0)_8) \cup (\Gamma_2'^{22,6} \oplus (v)_8) \cup (\Gamma_3'^{22,6} \oplus (s)_8) \cup (\Gamma_4'^{22,6} \oplus (c)_8)] \oplus \Gamma^{0,8}, \quad (3.7)$$

where $\Gamma_i'^{22,6}$ ($i = 1, \ldots, 4$) are $(22+6)$-dimensional conjugacy classes. This implies that, after reversing the bosonic string map, we obtain the toroidal compactifications of the ten-dimensional non-supersymmetric heterotic strings.

4 Classification of asymmetric orbifold models

Let us discuss the classification of asymmetric orbifold models. Since we are investigating the geometric interpretation of asymmetric orbifold models as toroidal compactifications, we classify the asymmetric orbifolds with the right-moving twist-untwist intertwining

currents. We first consider the choice of the momentum lattices. Unlike symmetric orbifolds models, the momentum lattices $\Gamma^{6,6}$ are severely restricted by the left-right asymmetric automorphisms. One of the known classes of such momentum lattices are given by

$$\Gamma^{6,6} = \{(p_L^i, p_R^i)|p_L^i, p_R^i \in \Lambda_W \text{ and } p_L^i - p_R^i \in \Lambda_R\}, \tag{4.1}$$

where Λ_W and Λ_R are the weight and root lattices of a simply-laced semisimple Lie algebra with the squared length of roots normalized to two [12]. The left- and right-moving rotation matrices U_L and U_R are taken to be the Weyl group elements [13] of the Lie algebra. Then the matrices U_L and U_R always satisfy the condition for the automorphism of $\Gamma^{6,6}$. Next, we consider the standard embeddings in the gauge degrees of freedom. Since the gauge group of the tachyon free non-supersymmetric strings is $SO(16) \times SO(16)$, we can embed the shift v_L^I in the $SO(6)$ subgroup of the first $SO(16)$. Let us denote the eigenvalues of U_L by $\{e^{i2\pi\zeta_L^a}, e^{-i2\pi\zeta_L^a}; a = 1, 2, 3\}$. Then the shift vector v_L^I is chosen as

$$v_L^I = (\zeta_L^a, 0^5; 0^8). \tag{4.2}$$

By the above choice of the shift vector, the first equation of the level matching conditions reduces to

$$N_\ell \sum_{a=1}^{3} \ell\zeta_L^a = 0 \mod 2. \tag{4.3}$$

With these level matching conditions, we classify the modular invariant asymmetric orbifold models.

To determine fermion currents on asymmetric orbifolds, we use the bosonic string map. We must investigate the full right-moving Kač-Moody algebras of asymmetric orbifolds. This is because, unlike symmetric orbifold case [14], there is no simple diagrammatical method for determining Kač-Moody algebras if there exist twist-untwist intertwining currents. The results of calculation are summarized in table 1. We present in table 1 the right-moving Kač-Moody algebras and the fermion currents of the asymmetric orbifold models. As we have seen above, the necessary and sufficient condition for the asymmetric orbifold models to be interpreted as toroidal compactifications of non-supersymmetric heterotic strings is that the fermion currents on the asymmetric orbifolds should generate $SO(8)$ Kač-Moody algebras. In table 1, we see that many

asymmetric orbifold models are geometrically interpreted as toroidal compactifications of non-supersymmetric heterotic strings.

5   Supersymmetric asymmetric orbifold models

We will present examples of the supersymmetric asymmetric orbifold models constructed from non-supersymmetric heterotic strings. We first consider the condition for obtaining massless gravitino states in the g^ℓ-sector ($\ell = 0, 1, \ldots, N-1$) of asymmetric orbifold models. The existence of a massless gravitino state will lead to the existence of a supersymmetry. Let us define the g^ℓ-invariant sublattice I_ℓ of $\Gamma^{6,6}$ by

$$I_\ell = \{(p_L^i, p_R^i) \in \Gamma^{6,6} | ((U_L^\ell)^{ij} p_L^j, (U_R^\ell)^{ij} p_R^j) = (p_L^i, p_R^i)\}. \tag{5.1}$$

The momentum (p_L^i, p_R^i) of the g^ℓ-twisted sector lies on the lattice I_ℓ^*, where I_ℓ^* is the dual lattice of I_ℓ. The number of degeneracy of the ground states in the g^ℓ-twisted sector is given by

$$n_\ell = \frac{\sqrt{\det'(\mathbf{1} - U_L^\ell) \det'(\mathbf{1} - U_R^\ell)}}{\text{vol}(I_\ell)}, \tag{5.2}$$

where $\det'$ is evaluated over the eigenvalues of U_L^ℓ and U_R^ℓ not equal to one and $\text{vol}(I_\ell)$ is the volume of the unit cell of the lattice I_ℓ. We shall denote the eigenvalues of U_R^ℓ by $\{e^{i2\pi\bar{\zeta}_\ell^a}, e^{-i2\pi\bar{\zeta}_\ell^a}; a = 1, 2, 3\}$. The mass formula in the g^ℓ-sector is given by

$$\frac{1}{8}m_L^2 = \frac{1}{2}\sum_{I=1}^{16}(p_L^I + \ell v_L^I)^2 + \frac{1}{2}\sum_{i=1}^{6}(p_L^i)^2 + \frac{1}{2}\sum_{a=1}^{3}\zeta_\ell^a(1 - \zeta_\ell^a) + N_L - 1, \tag{5.3}$$

$$\frac{1}{8}m_R^2 = \frac{1}{2}\sum_{i=1}^{6}(p_R^i)^2 + \frac{1}{2}\sum_{t=1}^{4}(p_R^t + \ell v_R^t)^2 + \frac{1}{2}\sum_{a=1}^{3}\bar{\zeta}_\ell^a(1 - \bar{\zeta}_\ell^a) + N_R - \frac{1}{2}, \tag{5.4}$$

where $(p_L^i, p_R^i) \in I_\ell^*$, $(p_L^I, p_R^t) \in (\Gamma_1^{16,0} \oplus (v)_4) \cup (\Gamma_2^{16,0} \oplus (0)_4) \cup (\Gamma_3^{16,0} \oplus (s)_4) \cup (\Gamma_4^{16,0} \oplus (c)_4)$, and N_L and N_R are the number operators of oscillators. In order to have massless spin 3/2 states, we must have $p_R^{t=4} = \pm 1/2$, $N_L = 1$, $U_L^\ell = \mathbf{1}$ and $p_L'^I \equiv p_L^I + \ell v_L^I = 0$ for $p_L^I \in \Gamma_3^{16,0} \cup \Gamma_4^{16,0}$, where the condition $N_L = 1$ must be satisfied by the oscillators of the space-time coordinates. It should be noted that $p_L'^I = 0$ for $p_L^I \in \Gamma_3^{16,0} \cup \Gamma_4^{16,0}$ never holds if we set $\ell = 0$ (untwisted sector) or $v_L^I = 0$ (no embedding). Let us denote the order of the left-moving shift (twist) as N_S (N_L). Then, from the mass formula we can check that there is no massless gravitino state for the orbifold models with $N_S = N_L$.

Examples of the models satisfying such a condition are the orbifold models with standard embeddings and symmetric orbifolds. Thus, we have to consider the asymmetric orbifold models with non-standard embeddings in order to obtain massless gravitino states.

We now construct examples of supersymmetric asymmetric orbifold models. We start with a toroidal compactification of $SO(16) \times SO(16)$ non-supersymmetric heterotic strings, where the momentum lattice $\Gamma^{6,6}$ is associated with a Lie algebra $(SU(3))^3$. The left- and right-twists of asymmetric orbifolds are taken to be the $\mathbf{Z}_3$-twist matrix U whose eigenvalues are given by $\{e^{2\pi i \zeta^a}, e^{-2\pi i \zeta^a}; a = 1, 2, 3\}$ with $\zeta^a = (1/3, 1/3, 2/3)$. We take the shift vector to be $v_L^I \in \Gamma_3^{16,0}$. For example, such shift vectors are given by $v_L^I = (1, 0^7; 1, 0^7)$ and $v_L^I = ((1/2)^8; (1/2)^8)$. The order of the shift is given by $N_S = 2$, and the level matching conditions are satisfied since $(v_L^I)^2 = 0 \mod 2$. The first example is the asymmetric $\mathbf{Z}_6$-orbifold model with $U_L = \mathbf{1}$ and $U_R = U$. Massless gravitino states may appear from $\ell = 1, 3, 5$ twisted sectors since the solution of $p_L'^I = 0$ exists for $\ell = 1, 3, 5$ if we set $p_L^I \in \Gamma_3^{16,0}$. In the $\ell = 1$ sector, from the massless condition we obtain $p_R^t = (-1/2, -1/2, -1/2, -1/2)$. The degeneracy of the ground states is given by $n_1 = 1$. This state is physical since all massless states in the first twisted sector are known to be physical [15]. In the $\ell = 3$ sector, we obtain $p_R'^t = (\pm 1/2, \pm 1/2, \pm 1/2, \pm 1/2)$, where the number of minus signs should be even. Since $U_L^3 = U_R^3 = \mathbf{1}$, the g operator in this sector will be given by $g = \exp[p_L'^I v_L^I - p_R'^t v_R^t - \frac{1}{2}\ell((v_L^I)^2 - (v_R^t)^2)]$. Thus, physical (i.e. $g = 1$) states are given by $p_R'^t = \pm(1/2, 1/2, -1/2, -1/2)$. In the $\ell = 5$ sector, we obtain $p_R^t = (-3/2, -3/2, -7/2, 1/2)$. The degeneracy of the ground states is given by $n_5 = 1$. This state is physical since all massless states in the g^5-twisted sector are physical. The existence of these massless states will lead to the existence of two gravitino states. Therefore, in this model we have $N = 2$ supersymmetry. The second example is the asymmetric $\mathbf{Z}_6$-orbifold model with $U_L = U$ and $U_R = \mathbf{1}$. From the mass formula, we see that gravitino states may appear from the $\ell = 3$ twisted sector. In the $\ell = 3$ sector, we obtain $p_R'^t = (\pm 1/2, \pm 1/2, \pm 1/2, \pm 1/2)$, where the number of minus signs should be even. The g operator takes the value $g = 1$ for all such massless states. Thus, the above states are all physical. The existence of these massless states will lead to the existence of four gravitino states. Therefore, in this model we have $N = 4$ supersymmetry. This model will possess the geometric interpretation as a

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Narain's toroidal compactification by the mechanism discussed in the previous paper [2]. The last example is the asymmetric $\mathbf{Z}_6$-orbifold model with $U_L = U_R = U$. We see that gravitino states may appear from the $\ell = 3$ twisted sector. In the $\ell = 3$ sector, we obtain $p_R'^t = (\pm 1/2, \pm 1/2, \pm 1/2, \pm 1/2)$, where the number of minus signs should be even. The g operator will be given by $g = \exp[p_L'^I v_L^I - p_R'^t v_R^t - \frac{1}{2}\ell((v_L^I)^2 - (v_R^t)^2)]$. Then physical states are given by $p_R'^t = \pm(1/2, 1/2, -1/2, -1/2)$. The existence of these massless states will lead to the existence of one gravitino state. Therefore, in this model we have $N = 1$ supersymmetry.

6 Conclusion

In this paper we have constructed asymmetric orbifold models which are based on non-supersymmetric heterotic strings. We have shown that a simple condition in terms of fermion currents is necessary and sufficient for our class of asymmetric orbifolds to be interpreted as the toroidal compactifications of non-supersymmetric heterotic strings. We have made a systematic classification of the asymmetric $\mathbf{Z}_N$-orbifold models with standard embeddings and obtained many asymmetric orbifold models which are geometrically interpreted as toroidal compactifications of non-supersymmetric strings. We have also discussed the supersymmetric asymmetric orbifold models constructed from non-supersymmetric heterotic strings. We have investigated the condition for obtaining supersymmetric $\mathbf{Z}_N$-orbifold models from non-supersymmetric heterotic strings and have shown that in order to obtain supersymmetric models we must consider asymmetric orbifold models with non-standard embeddings. We have constructed the examples of the $N = 1$, $N = 2$ and $N = 4$ asymmetric orbifold models of non-supersymmetric heterotic strings. It would be of interest to investigate other examples of such supersymmetric models constructed from non-supersymmetric heterotic strings.

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Table 1: List of asymmetric $\mathbf{Z}_N$-orbifold models of non-supersymmetric heterotic strings with the right-moving twist-untwist intertwining currents.

Right-moving Kač-Moody algebra	Fermion current	Number of models
$(SU(2))^8$	$SO(4)$	4
$(SU(2))^6 \times SU(3)$	$SO(4)$	11
$(SU(2))^6 \times (U(1))^3$	$SO(2)$	3
$(SU(2))^5 \times SU(4)$	$SO(4)$	4
$(SU(2))^5 \times (U(1))^2$	$SO(6)$	1
$(SU(2))^4 \times SU(3)$	$SO(8)$	1
$(SU(2))^4 \times SO(8)$	$SO(4)$	14
$(SU(2))^4 \times (U(1))^5$	$SO(2)$	3
$(SU(2))^4 \times (U(1))^4$	$SO(4)$	3
$(SU(2))^3 \times SU(3) \times (U(1))^2$	$SO(6)$	3
$(SU(2))^3 \times SU(4)$	$SO(8)$	1
$(SU(2))^3 \times SU(6)$	$SO(4)$	11
$(SU(2))^2 \times (SU(3))^2 \times (U(1))^2$	$SO(4)$	1
$(SU(2))^2 \times (SU(3))^2$	$SO(8)$	2
$(SU(2))^2 \times SU(3) \times (U(1))^4$	$SO(4)$	1
$(SU(2))^2 \times SU(4) \times (U(1))^2$	$SO(6)$	1
$(SU(2))^2 \times SU(5)$	$SO(8)$	4
$(SU(2))^2 \times SO(8) \times (U(1))^3$	$SO(2)$	2
$(SU(2))^2 \times SO(8)$	$SO(8)$	8
$SU(2) \times SU(3) \times SU(4)$	$SO(8)$	3
$SU(2) \times SU(6)$	$SO(8)$	7
$SU(2) \times SO(8) \times (U(1))^3$	$SO(4)$	5
$SU(2) \times SO(8) \times (U(1))^2$	$SO(6)$	2
$SU(2) \times SO(10)$	$SO(8)$	17
$(SU(3))^4 \times U(1)$	$SO(2)$	15
$(SU(3))^3$	$SO(8)$	2
$SU(3) \times SU(5)$	$SO(8)$	11
$SU(3) \times SO(8)$	$SO(8)$	21
$SU(3) \times (U(1))^6$	$SO(4)$	1
$(SU(4))^2$	$SO(8)$	2
$SU(5) \times (U(1))^3$	$SO(6)$	3
$SU(6) \times (U(1))^2$	$SO(6)$	4
$SU(7)$	$SO(8)$	14
$SO(8) \times (U(1))^3$	$SO(6)$	6
$SO(12)$	$SO(8)$	45
E_6	$SO(8)$	47